

Spring 2019 Math 245 Exam 2

Please read the following directions:

Please write legibly, with plenty of white space. Please fill out the box above as legibly as possible. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

- a. Nonconstructive Existence Theorem

- b. Proof by Shifted Induction

- c. Proof by Strong Induction

Problem 2. Carefully define the following terms:

- a. recurrence

- b. big Omega (Ω)

- c. big Theta (Θ)

Problem 3. Let $a, b \in \mathbb{Z}$ with $b \geq 1$. Use minimum element induction to prove that there exist integers q, r with $a = bq + r$ and $0 < r \leq b$.

Problem 4. Let $x \in \mathbb{R}$. Prove that $\lceil x \rceil$ is unique; i.e., prove there is at most one integer n with $n - 1 < x \leq n$.

Problem 5. Let F_n denote the Fibonacci numbers. Prove that for all $n \in \mathbb{N}$, we have $F_{2n} = \sum_{i=0}^{n-1} F_{2i+1}$.

Problem 6. Prove that for all $n \in \mathbb{N}$ with $n \geq 4$, we have $n! > 2^n$.

Problem 7. Let $a_n = n^{1.9} + n^2$. Prove that $a_n = O(n^2)$.

Problem 8. Solve the recurrence given by $a_0 = 2, a_1 = 6, a_n = 5a_{n-1} - 6a_{n-2}$ ($n \geq 2$).

Problem 9. Prove that for all real x , we have $|x - 1| + |x + 2| \geq 3$.

Problem 10. Let $x \in \mathbb{R}$. Prove that $\lfloor x + \frac{1}{2} \rfloor = \lfloor x \rfloor$ if and only if $x - \lfloor x \rfloor < \frac{1}{2}$.